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Numerical analysis of morphing corrugated plates

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Abstract

In this paper a numerical model for investigating the moment-rotation response of corrugated plates is presented. In particular, the effect of the geometry of the plate on the bending response is considered. Results are compared with a simplified theoretical model recently appeared in the literature. Combining geometrical effects and prestress, corrugated plates can become multistable forming the basis of new morphing structures.

Keywords: morphing structures; corrugated plates

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1. Introduction

Conventional structures are usually designed to maintain a single shape throughout their design life; however more often in the engineering practice is useful to have some or more structural members changing their shape under operative conditions. Corrugated plates constitute the basis to create new morphing structures since they can become multistable combining geometrical effects and prestress, Kebadze et al. (2004). Recently, a simplified analytical model to study the bending response of thin corrugated shells has appeared in the literature, Norman et al. (2008).

The aim of the paper is to investigate geometrical effects of some corrugated plates on the bending response by means of an enriched non linear FE model. Some results are compared with analytical ones given by the theoretical model, showing that a numerical analysis is important in order to distil some key features of the bending response, such as the peak and the propagating moments.

Nomenclature

D	flexural rigidity
E, ν	Young's modulus, Poisson's ratio
m	bending moment per unit length
M	bending moment

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$M_+^{\max}$, $M_-^{\max}$	positive and negative peak moments
M_+^*, M_-^*	positive and negative fold propagation moments
t, ϕ, λ	corrugated plate thickness, subtended angle, corrugation wave length

2. Bending response

A corrugated plate of thickness t, length L and subtending an angle equal to ϕ is sketched in Fig. 1. In this study, plates with 6 half-waves are considered. The 6 half-waves plate presents 3 corrugations with a downward curvature and 2 corrugations with an upward curvature and two half corrugations at the edges.

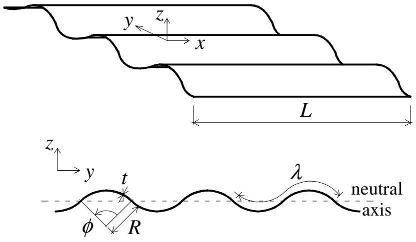

Fig. 1. Schematic representation of the 6 half-waves corrugated plate

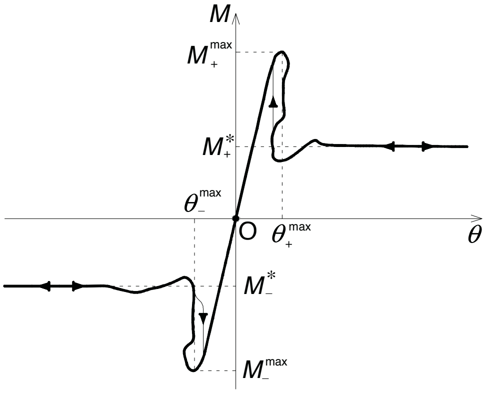

Fig. 2. Schematic moment (M) - end rotation (θ) diagram

The plate is bent along the y-axis such that the ends are rigidly encased. The resulting bending response M-θ is represented in Fig. 2, where θ is the relative rotation between the ends. As it can be inferred from the figure, the behaviour of the corrugated plate is highly non linear. For small rotations, initially M varies linearly with θ, both for positive and negative moments. As the moment increases, the cross-section of the plate begins to flatten in the middle. The moment reaches a maximum indicated with $M_+^{\max}$. Then, the deformation localises in a longitudinally curved region while the moment decreases quickly. The snap-through behaviour is not 'neat': usually it starts from the flattening of the central corrugation, and then also the others begin to localize and they snap through. Then, the moment reaches a minimum, but after, in order for the rotation to increase, the moment has to increase again. The moment attains an approximately constant value, called propagating moment M_+^*. When the direction of turning is reversed (see the black arrows in the figure), the same path is followed, until a certain value of the rotation, which corresponds to a value of M lower than that attained upon loading. Then, the same path is followed till the moment reaches zero value.

With the application of a negative moment, the free edges are in compression, thus, before localisation, local buckling of the edges is observed. The moment shows the same behaviour as when a positive moment is applied, remaining constant as θ is further increased. The absolute value of the propagating moment M_{-}^{*} is almost the same as the positive one. When θ is decreased, the unloading path practically coincides with the loading path until a certain value of rotation is reached, which corresponds to a value of M lower (in absolute value) than that attained upon loading. Again, the behaviour under positive and negative moment during the unloading path is almost symmetric.

3. Numerical analysis

Figures 3(a) and (b) present some numerical results for different lengths of plate characterized by the same width (6 half-waves), with subtended angle $\phi = 43^o$, radius of curvature R = 20 mm and thickness t equal to 0.2 mm and 0.3 mm, respectively. The mechanical properties of the plates are: Young's modulus E equal to 131 GPa and Poisson's ratio v equal to 0.3. Numerical analysis is conducted by means of the software package ABAQUS, Hibbit et al. (1997). For convenience, the applied moment M is non-dimensionalised by setting equal to $m / D\kappa_{T0}$, where m is the bending moment per unit length, D is the flexural rigidity and κ_{T0} is the initial corrugation curvature.

As it can be seen, the shortest plate has the largest peak moment and, as the length increases, the peak moment decreases and tends towards a constant value. The longer the corrugated plate becomes, the smaller is the influence of the constraint applied by the rigid ends to the localisation which follows snap-through. Thus, the corrugated plate becomes softer, and the initiation of the fold takes place at a lower bending moment. The post-buckled behaviour remains essentially the same for all the lengths. The initial slope is steeper for a decreasing length of the plate. This behaviour has been noted also for tape-springs, Seffen and Pellegrino (1999). Further insight is given also in Figs. 4(a) and (b), where the non-dimensional peak moments $m_{+}^{\max} / D\kappa_{T0}$ for the different geometries are collected and compared with analytical results. In the figure also results for 10 half-wave plates, when available, are reported (dotted line). Above a certain value of the ratio between the length and the projected width of the plate (between 3 and 4), the rigid ends lose their effects on the bending behaviour of the corrugated plate, that is the peak moment is not influenced by the length of the plate, as noted earlier.

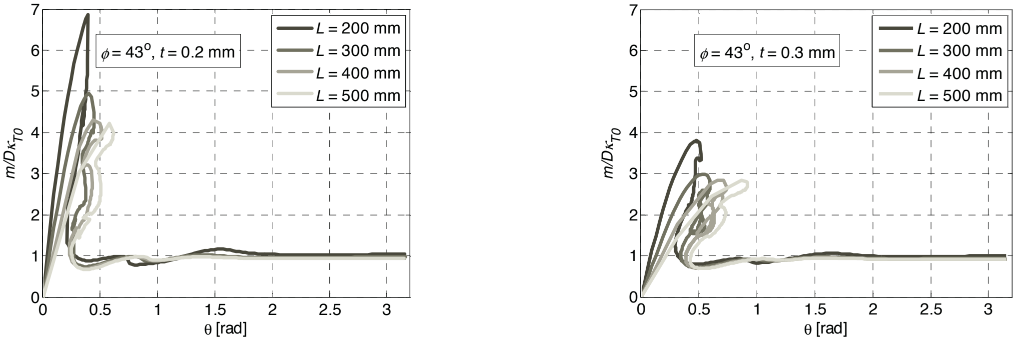

Fig. 3. Non-dimensionalised moment-rotation relationship for 6 half-waves corrugated plates with subtending angle ϕ = 43° and thickness (a) t = 0.2 mm; (b) t = 0.3 mm

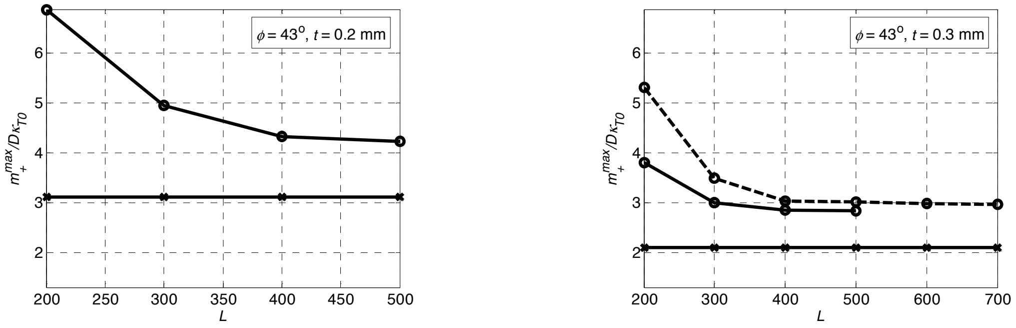

Fig. 4. Plots of non-dimensional peak moment $m^{max}{}_{+}/D\kappa_{T0}$ against L for corrugated plates with subtending angle $\phi = 43^o$ and thickness (a) $t = 0.2$ mm; (b) $t = 0.3$ mm. Results are plotted as follows: solid line with "o" corresponds to numerical results for 6 half-waves plate, dashed line with "o" corresponds to numerical results for 10 half-waves plate and solid line with "x" corresponds to analytical results

Moreover, the analytical model assumes that flattening is a uniform process in which the corrugations remain as circular arcs: this may not be true in practice especially when the localisation, which precedes snap-through, stems from the corrugations flattening in a sequential manner across the section. Differences can be also due to the fact that in the analytical model only bending stresses along the corrugations are considered. In reality also stresses that act along the transverse axis are present and are not negligible. In fact, the two stresses have a comparable magnitude. As the plate widens, the peak moments are larger but they tend towards the same values of the 6 half-wave plates as the lengths of the plate increases.

4. Conclusions

In this study, some important features of the bending response of corrugated plates are investigated by means of a finite element model. Differently from the simplified analytical model presented in the literature, the numerical model is able to take into account some important aspects of behaviour such as end effects and geometrical effects. Some comparisons between the analytical model and the enriched numerical model of the corrugated plate has been conducted. Results have shown on one hand, the ability of the analytical model to capture the overall behaviour of corrugated plates. On the other hand, numerical analysis is able to distill the key features, as peak moments, that characterize the bending behaviour of the plate that have to be known for a robust design.

Acknowledgements

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